[19] "IRE Standards on Antennas and Waveguides," PROC. IRE, vol. 47, pp. 568–582; April, 1959. See especially p. 581.
[20] B. Bramham, "A convenient transformer for matching coaxial lines," *Electronic Engrg.*, vol. 33, pp. 42–44; January, 1961.
[21] *Reference Data for Radio Engineers*, 4th ed., pp. 193–198, ITT Corporation, New York, N. Y.; 1956.
[22] D. R. J. White, "Charts simplify passive L-C filter design," *Electronic*, vol. 30, pp. 160–163, December 1, 1957; and "Band-pass filter design technique," *Electronic*, vol. 31, pp. 79–81; January 3, 1958.
[23] G. C. Southworth, "Principles and Application of Waveguide Transmission," D. Van Nostrand Co., Inc., New York, N. Y.; 1950.
[24] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beamwidth and side-lobe level," PROC. IRE, vol. 34, pp. 335–348; June, 1946. See also "Discussion" in PROC. IRE, vol. 35, pp. 489–492; May, 1957.
[25] L. B. Brown and G. A. Sharp, "Tchebyscheff Antenna Distribution, Beamwidth, and Gain Tables," Naval Ordnance Laboratory, Corona, Calif. NAVORD Report 4629, NOLC Report 383; February, 1958.
[26] M. L. Reuss, Jr., "Some Design Considerations Concerning Linear Arrays Having Dolph-Tchebyscheff Amplitude Distributions " NRL Rept. 5240, No. AD 212 621; February, 1959.
[27] G. J. Van der Maas, "A simplified calculation for Dolph-Tchebyscheff arrays," *J. Appl. Phys.*, vol. 24, p. 1250; September, 1953.
[28] G L. Matthaei, *et al.*, "Design Criteria for Microwave Filters and Coupling Structures," Stanford Research Institute, Menlo Park, Calif., Final Rept., SRI Project 2326, Contract No. DA 36-039 SC-74862, Tables, pp. 202 and 203; January, 1961. Also, Tech. Note 4 on same project, Tables, pp. 19, 20.)

On Inhomogeneous Quarter-Wave Transformers:

See [1], [7], [8], [16], and

[29] L. Young, "Spin matrix exponentials and transmission matrices," *Quart. Appl. Math.*, vol. 19, pp. 25–30; April, 1961.

On Half-Wave Filters with Periodic Structure:

An optical situation frequently encountered where the same two materials *a* and *b* are used alternately to form a multilayer *abab* . . . is treated by

[30] K. D. Mielenz, "Use of Chebychev polynomials in thin film computations," *J. Res. NBS*, vol. 63A, pp. 297–300; November–December 1959. [There is a misprint in (17b): the lower left element in the matrix should be $S_{m-1}a_{21}$.]

On Nonsynchronous Transformers:

An example is given in [20]. Another case is considered by

[31] J. T. Cox, G. Hass and R. F. Rowntree, "Two-layer anti-reflection coatings for glass in the near infra-red," *Vacuum*, vol. 4, pp. 445–455; October, 1954.

On Effect of Dissipation Losses:

General formulas for quarter-wave transformers and half-wave filters when the dissipation loss is small are given by

[32] L. Young, "Prediction of absorption loss in multilayer interference filters," *J. Opt. Soc. Am.*, vol. 52, pp. 753–761; July, 1962.

Other formulas based on lumped-constant circuits with small dissipation loss but also applicable to narrow-band selective transmission-line filter with small dissipation loss are given by

[33] S. B. Cohn, "Dissipation loss in multiple-coupled-resonator filters," PROC IRE, vol. 47, pp. 1342–1348; August, 1959.

On Power-Handling Capability:

The power-handling capacity of filters based on quarter-wave transformer prototypes may be estimated from

[34] L. Young, "Peak internal fields in direct-coupled-cavity filters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 612–616; November, 1960.

and for filters based on the lumped-constant low-pass prototype, from

[35] S. B. Cohn, "Design considerations for high-power microwave filters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 149–153; January, 1959.

In-Line Waveguide Calorimeter for High-Power Measurement*

M. MICHAEL BRADY†, MEMBER, IRE

***Summary*—The static in-line calorimeter measures the temperature rise in the walls of a waveguide caused by the attenuation of microwave power flowing through the waveguide. It is simple and inexpensive and can be constructed so that it will fit on waveguide already existing in a microwave system. The device should be reliable because it uses no active circuitry. In addition, few mechanical problems are encountered in its use because the existing waveguide need not be altered. The theory of the device is developed, and two experimental *S*-band calorimeters using stainless steel waveguide and resistance-wire bridge temperature indicators are described. The measured sensitivity and time constant for both units fall within the experimental error of confirming the theoretically predicted figures.**

* Received April 26, 1962. This work was supported by the U. S. A.E.C. uncer Contract AT(04-3)-21, Proj. Agreement No. 1.
† Stanford Linear Accelerator Center, Stanford University, Stanford, Calif.

INTRODUCTION

THE HIGH-POWER measurement or monitoring schemes presently available, if not complete absorption devices, are usually reduced-signal sampling devices in which a low-power meter is used. In most low-level measurement schemes, however, the background noise (ambient temperature fluctuations in the case of the bolometer power meter) often determines the ultimate resolution of the device. In a high-power measurement system, on the other hand, the power level present most often completely masks the low-power background noise. It may then be less desirable to sample a high-power signal through a directional coupler and to attenuate the sampled signal until it can be read with conventional low-power meters than to use a measurement scheme that responds to the high-power signal directly.

The device described in this paper is one that responds directly to a high-power microwave signal. It consists of a static in-line dry calorimeter, a device that measures the temperature rise in the walls of a waveguide caused by the attenuation of power flowing through the waveguide. Its basic simplicity combined with its adaptability to different waveguide configurations makes it reliable and inexpensive. In addition, few mechanical problems are encountered in its use because the existing waveguide of the measured system need not be altered.

In this paper the first-order theory of the in-line calorimeter is developed. The predictions of the theory are in reasonably close agreement with the experimental results that were obtained from two S-band models of a prototype calorimeter. As expected from a calorimetric device, the in-line calorimeter has a relatively long time constant, varying from less than half a minute to slightly over ten minutes for the practical calorimeters considered. The power sensitivity for an experimental S-band model 7.6 cm long was 12.5 mv output per volt into bridge circuit per kilowatt of microwave power flowing through the measured waveguide section, while for a model 14.98 cm long the corresponding figure was 47 mv. The error in these experimental figures, which was 10 per cent, extends the experimental figures to cover the theoretically predicted figures for sensitivity. The minimum resolution of the experimental calorimeters, which were designed to measure average power levels of a half to several kilowatts, was found to be less than half a watt.

The In-Line Calorimeter

If a section of waveguide of known attenuation is provided with heat sinks at either end and is well insulated throughout its length, then absorption of power in the walls of the waveguide will cause the center to be elevated in temperature above the ends. As will be shown, the temperature difference between the center and the ends of the section is directly proportional to the power absorbed and thus to the power transmitted through the section. Such a waveguide section, along with its auxiliary temperature measuring apparatus, comprises an in-line dry calorimeter capable of indicating the average power level in a waveguide. Once the characteristics of such a calorimeter are known, it could well serve as a transfer power standard, for its indication is derived entirely from directly measurable quantities.

In 1942 a similar type of calorimeter was proposed in which portions of the wall of a section of waveguide had been replaced with thin constantin sheets.[1] The device did not use heat sinks and was thus sensitive to temperature gradients along the waveguide. Further practical difficulties in calibration limited the usefulness of the device, and it was apparently discarded.

[1] M. H. Johnson, "Microwave Wattmeter," Radiation Lab., M.I.T., Cambridge, Mass., Rept. No. 53-8; November 8, 1942.

The Static Waveguide Calorimeter Equations

For ease of computation, the calorimeter will be assumed to be adequately represented by a rod of length and volume equal to the actual length and metal volume of the waveguide in the actual calorimeter, as shown in Fig. 1. The dissipation of microwave power in the walls of the waveguide will be treated as a uniformly distributed source of power. The dissipation of power normally occurs only within the skin depth of the waveguide, and the remainder of the metal present acts as a heat sink. If the waveguide is constructed of a metal having a thermal conductivity far higher than that of its surroundings and is insulated to minimize convection and radiation heat losses, then the rod model should be a quite accurate representation of the calorimeter.

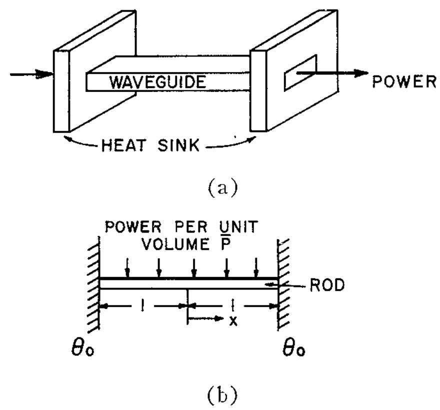


Fig. 1—(a) In-line calorimeter. (b) Rod approximation.

The equations governing the flow of heat in the rod are the conventional heat-flow equations, modified to include the continuous addition of energy along the rod. If an external source adds an amount of energy per unit volume along the rod at a rate of $\overline{P}$, then, because the volume contains neither sources nor sinks of heat, the conventional heat-flow equation is modified to read

$$K\nabla^2\theta - s\rho\dot{\theta} + \overline{P} = 0 \qquad (1)$$

where θ is the temperature in degrees Celsius, s is the thermal capacity in joules per degree Celsius per gram, ρ is the density in grams per cubic centimeter, and K is the thermal conductivity in watts per degree Celsius per centimeter. For the one-dimensional case this becomes

$$\frac{\partial^2\theta}{\partial x^2} - \frac{s\rho}{K}\frac{\partial\theta}{\partial t} = -\frac{\overline{P}}{K}. \qquad (2)$$

Eq. (2) can be solved by considering separately the steady-state equation for which $\partial\theta/\partial t = 0$ and the transient equation for which $\overline{P} = 0$. The steady-state solution can be obtained by applying the boundary conditions that $\partial\theta/\partial x = 0$ when $x = 0$, and $\theta = 0$ when $x = \pm l$. The result is a parabolic solution of the form

$$\theta = \frac{\overline{P}}{2K}(l^2 - x^2) \tag{3}$$

where θ is the temperature above the end temperature because of the specification of zero as the arbitrary end temperature. The transient solution can be obtained by applying the appropriate boundary conditions to the transient portion of (2). It is simpler, however, to demand that the transient solution be equal in magnitude and opposite in sign to the steady-state solution for zero time, and zero for infinite time. The net result is that the entire rod is at zero (reference) temperature at $t=0$ and assumes the steady-state solution of (3) for t infinite. The method of attack will be to study the problem of the temperature distribution $\theta(x, t)$ in a rod when the initial temperature distribution $\theta(x, 0)$ is a parabola of the form of (3). The transient portion of (2) can be written

$$\frac{\partial\theta}{\partial t} = \frac{K}{s\rho}\frac{\partial^2\theta}{\partial x^2}. \tag{4}$$

The initial temperature distribution $\theta(x, 0)$ of (3) can be expanded in a Fourier series containing cosine terms only because the x coordinates chosen for the parabola make it an even function of x. The Fourier series solution for $\theta(x, t)$ can be written as

$$\theta(x, t) = \sum_{n=1}^{\infty} A_n \exp\left[-\left(\frac{n\pi}{2l}\right)^2 \frac{K}{s\rho} t\right] \cos\left(\frac{n\pi x}{2l}\right). \tag{5}$$

Taking the first and second partial derivatives of the right-hand side of (5) with respect to t shows that (4) is satisfied; thus (5) is a valid form for $\theta(x, t)$. Now for $t=0$, $\theta(x, 0)$ must be equal to the negative of the steady-state distribution

$$\sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{2l}\right) = -\frac{\overline{P}}{2K}(l^2 - x^2)$$

where the coefficients A_n are evaluated as

$$A_n = \frac{1}{l}\int_{-l}^{l} \theta(x, 0)\cos\left(\frac{n\pi x}{2l}\right) dx.$$

By straightforward evaluation the A_n's are found to be

$$A_n = \frac{16\overline{P}l^2(-1)^n}{Kn^3\pi^3} \qquad \text{for } n \geq 0. \tag{6}$$

Thus the complete solution for the temperature distribution in the rod as a function of both distance along the rod and time is

$$\theta(x, t) = \frac{\overline{P}}{2K}\left\{l^2 - x^2 - \frac{32l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3} \cdot \exp\left[-\left(\frac{(2n+1)\pi}{2l}\right)^2 \frac{K}{s\rho} t\right]\cos\left(\frac{n\pi x}{2l}\right)\right\}. \tag{7}$$

The solution of (7) for $t=0$ should reduce to zero. As a check, $\theta(0, 0)$ is

$$\theta(0, 0) = \frac{\overline{P}l^2}{2K}\left\{1 - \frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\right\}.$$

This will reduce to zero if the summation is equal to the quantity $\pi^3/32$. The summation is similar to an Euler polynomial which has the form[2]

$$E_{2r} = \frac{(-1)^r 2r!(2)^{2r+2}}{(\pi)^{2r+1}}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^{2r+1}}$$

where the first four Euler numbers are: $E_0=1$, $E_2=-1$, $E_4=5$, $E_6=-61$. Thus for $r=1$, E_2 is

$$E_2 = -1 = -\frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}.$$

Thus the summation in question is indeed equal to $\pi^3/32$, and $\theta(0, 0)$ reduces to zero. Eq. (7) may then be regarded as the complete solution for the temperature distribution in the calorimeter. The actual calorimeter is arranged to measure the temperature rise $\theta(0, t)$ of the center of the waveguide between the heat sinks, which is given by

$$\theta(0, t) = \frac{\overline{P}l^2}{2K}\left\{1 - \frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3} \cdot \exp\left[-\frac{(2n+1)^3 t}{\tau}\right]\right\}, \tag{8}$$

where $\overline{P}$ is the power per unit waveguide metal volume absorbed by the calorimeter, l is the half length of the waveguide between heat sinks, and $\tau = 4l^2 s\rho/\pi^2 K$ has the dimensions of time and thus is a kind of time constant. This time constant may, for S-band waveguides of common metals 6 cm in length, fall in the range of tens of seconds to several minutes. As can be seen from (8) the first term of the summation for $n=0$ will dominate, for the coefficients of the terms of the summation are the coefficients of the cosine terms of the series representing the steady-state parabolic distribution given by (3), and a parabola of small amplitude is fairly well approximated by the first term of its representative Fourier cosine series.

Both the time constant and the total temperature rise at the center of the calorimeter are seen to depend on the square of the half-length l. The design of a given calorimeter must then be a compromise between a long body for sensitivity and a short body for a fast rise time. The eventual temperature rise at the center of the calorime-

[2] Bateman Manuscript Proj. of the Calif. Inst. Tech., "Higher Transcendental Functions," McGraw-Hill Book Co., Inc, New York, N. Y., vol. 1, p. 42; 1953.

TABLE I
CHARACTERISTICS OF VARIOUS METALS

Material	s joules/°C-gm	ρ gm/cm³	K watts/°C-cm	α/α_{Cu}*	η °C-cm³/joules	$\theta(0, \infty)/\theta(0, \infty)_{Cu}$
Aluminum	0.896	2.69	2.135	1.28	0.511	2.31
Brass (yellow)	0.364	8.56	1.08	2.0	0.397	7.07
Constantin	0.418	8.88	0.226	4.97	0.322	85.6
Copper	0.38	8.89	3.82	1.00	0.361	1.00
Gold (drawn)	0.129	19.26	2.93	1.19	0.495	1.56
Nickel	0.440	8.70	0.595	1.88	0.323	12.12
Platinum	0.134	21.37	0.70	2.41	0.432	13.31
Silver	0.234	10.60	4.22	0.97	0.498	0.88
Steel, Carbon	0.482	7.87	0.594	2.76	0.325	17.92
Steel, 304 stainless	0.502	7.93	0.151	7.0	0.311	179.5

* Computed using square roots of ratios of resistivities.

ter as t approaches infinity is given by

$$\theta(0, \infty) = \frac{\overline{P}l^2}{2K} \cdot \tag{9}$$

It is interesting to note that the ratio of the calorimeter steady-state temperature rise per unit of input energy from (9) to the time constant of (8) is dependent only on the product of the thermal capacity s and density ρ.

$$\frac{\theta(0, \infty)/\overline{P}}{\tau} = \frac{\pi^2}{8s\rho} \equiv \eta \tag{10}$$

where η has the dimensions of temperature divided by energy per unit volume.

Because the product of the thermal capacity s and the density ρ is fairly constant for metals, η will be roughly the same for any metal, as illustrated by Table I (η for copper is 0.361). This, then, indicates that for a given absorbed power per unit volume, the ratio of expected temperature rise to time-constant is approximately constant.

Effect of a Nonzero Reflection Coefficient on Calorimeter Indication

Even if the calorimeter itself is perfectly matched, a standing wave of the H field in the waveguide will result in a standing wave of waveguide wall current. Since the power loss to the waveguide walls is proportional to the square of the wall current, a standing wave of power dissipated in the waveguide walls will exist. The calorimeter might then be expected to respond to this pattern in some way other than simply reading the sum of the incident and reflected power passing through the waveguide section.

Eq. (9), which expresses the temperature rise at the center of the calorimeter caused by the absorption of power in the walls of the waveguide, was based on the assumption that the H field, and thus the power absorbed in the waveguide walls, was uniform along the axis of the waveguide. This, of course, implies that the load reflection coefficient is zero. If, however, the load reflection coefficient is not zero, and there exists a standing-wave pattern of electric and magnetic fields within the waveguide, then the power dissipated in the walls of the waveguide will not be uniform along the axis of the calorimeter. In what follows, the absorbed-power standing-wave pattern will be assumed to be directly proportional to the square of the wall current standing-wave pattern and thus to the square of the H field standing-wave pattern, as shown in Fig. 2.

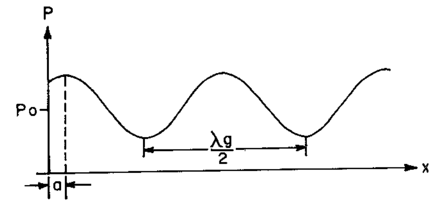


Fig. 2—Standing-wave pattern of power in a waveguide.

The waveguide will be assumed to have an attenuation that is small enough so that the ideal transmission-line equations describe the field distributions within the waveguide with negligible error. The power P absorbed in the waveguide walls is a function of the reflection coefficient Γ, the incident power P_i, the reflected power P_r, and an assumed random phase of the pattern ϕ. If the sum of P_i and P_r is set equal to a nominal power, $P_0 = P_i + P_r$, then P is

$$P = P_0\left[1 + \frac{2\Gamma}{1+\Gamma^2}\cos\frac{4\pi(x-\phi)}{\lambda_g}\right]. \tag{11}$$

Eq. (11) expresses the relationship between absorbed waveguide power P and axial displacement x. Thus the steady-state part of (2) is modified as follows:

$$\frac{\partial^2\theta}{\partial x^2} = -\frac{\overline{P}_0}{K}\left[1 + \frac{2\Gamma}{1+\Gamma^2}\cos\frac{4\pi(x-\phi)}{\lambda_g}\right]. \tag{12}$$

Integrating (12) twice with respect to x results in an equation with two unknown constants. Using the boundary conditions of $\theta = 0$ at $x = +l$ and $x = -l$ separately, two equations involving the unknown constants can be obtained; the constants can then be obtained algebraically. The complete solution of (12) for $x=0$ is then

$$\theta(0, t) = \frac{\overline{P}_0 l^2}{2K}\left\{1 + \frac{\lambda_g^2}{8\pi^2 l^2}\frac{2\Gamma}{1+\Gamma^2}\left(\cos\frac{4\pi\phi}{\lambda_g}\right)\cdot\left(1-\cos\frac{4\pi l}{\lambda_g}\right)\right\}. \quad (13)$$

Now defining the nominal center temperature rise according to (9) as $\theta_0 = \overline{P}_0 l^2/2K$, normalizing the length l and phase ϕ such that $L = 2\pi l/\lambda_g$, and $\Phi = 2\pi a/\lambda_g$, and defining the per-unit error as

$$\epsilon = [\theta(0, \infty) - \theta_0(0, \infty)]/\theta_0(0, \infty)$$

results in a simple error expression

$$\epsilon = \frac{2}{L^2}\frac{\Gamma}{1+\Gamma^2}\cos 2\Phi \sin^2 L.$$

The largest error would be for $\Phi = n\pi$ (n an integer), so the absolute value of the error is bounded

$$|\epsilon| \le \left(\frac{\sin L}{L}\right)^2 \frac{2|\Gamma|}{1+|\Gamma|^2}. \quad (14)$$

Eq. (14) may easily be plotted, since $\sin L/L$ is well tabulated and since the error $|\epsilon|$ as a function of length L with reflection coefficient $|\Gamma|$ as a parameter is of interest (see Fig. 3).

It should be noted that the error $|\epsilon|$ expresses the extent to which the calorimeter does not read the sum of the incident and reflected powers passing through its waveguide section. If the error is zero, the calorimeter simply reads the sum of P_i and P_r; the reflection coefficient must then be known in order to extract either P_i or P_r from the calorimeter reading. In order for the error $|\epsilon|$ to vanish, the normalized length L must be a multiple of π, or the half-length l must be an integral number of half-waveguide wavelengths, $l = n\lambda_g/2$, as illustrated by Fig. 3. In this case (13) reduces to (9), and the center temperature rise and thus the output indication is dependent only on the sum of the powers passing through the calorimeter.

Calorimeter Characteristics

Provided that the length of the calorimeter is chosen to be an integral number of waveguide wavelengths, in accordance with Fig. 3, the steady-state temperature rise at the center of the calorimeter is

$$\theta(0, \infty) = \frac{\overline{P} l^2}{2K}.$$

But $\overline{P}$ is the power per unit volume dissipated in the waveguide material; the length of the calorimeter is $2l$; if the cross-sectional metal area is a, the temperature

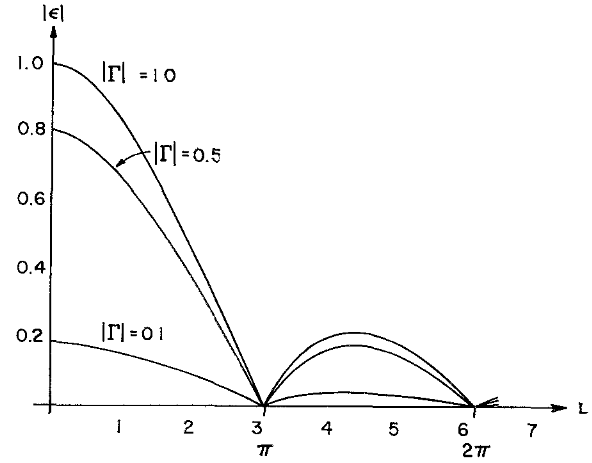


Fig. 3—Maximum error $|\epsilon|$ vs normalized length L.

rise due to the total absorbed power P is

$$\theta(0, \infty) = \frac{Pl}{4Ka}.$$

If the waveguide has an attenuation of α db per unit length, and if its length is short so that the average power leaving (P_{out}) is essentially the same as the average power entering (P_{in}), then P may be written

$$P = P_{\text{in}}(1 - 10^{-0.2l\alpha})$$

which gives

$$\theta(0, \infty) = \frac{lP_{\text{in}}}{4Ka}[1 - 10^{-0.2l\alpha}].$$

If the total attenuation through the waveguide section is very small, that is, if $0.2l\alpha \ll 1$, then $\theta(0, \infty)$ may be approximated by expanding $(1 - 10^{-0.2l\alpha})$ in a series and taking the first term

$$\theta(0, \infty) \approx \frac{0.115 l^2 P_{\text{in}}\alpha}{aK}. \quad (15)$$

Eq. (15) gives the temperature rise to be expected in the middle of a calorimeter constructed of given metal of given dimensions.

The selection of a material for the waveguide section of a specific calorimeter is based on a compromise among several factors. The major factor of concern is, of course, the temperature rise that can be expected per unit of microwave power flowing through the waveguide section; (15) can be used to predict $\theta(0, \infty)$ for any given calorimeter configuration. It is convenient to compare various materials using one as a reference. If copper is chosen as a reference, then $\theta(0, \infty)$ for various metals may be tabulated as a function of $\theta(0, \infty)$ for copper. From (15)

$$\frac{\theta(0, \infty)}{\theta(0, \infty)_{\text{Cu}}} = \frac{K_{\text{Cu}}}{K}\frac{\alpha}{\alpha_{\text{Cu}}}. \quad (16)$$

The various parameters for several metals are tabulated in Table I, along with the results of (16) for each case.

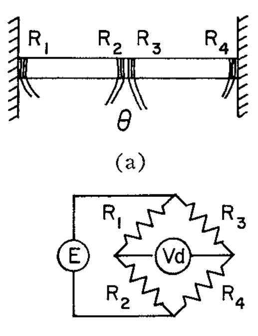
Temperature Indication

Given a specific temperature difference between the center and the ends of the calorimeter, some method must be used to give an indication of this difference so that the waveguide power may be measured. Two methods immediately suggest themselves: a resistance-wire bridge and a thermopile. The resistance-wire bridge has the advantage that as large a signal as desired can be obtained; it was therefore selected as the method of indication.

If four identical temperature-sensitive resistances are placed in contact with the waveguide and connected in a bridge circuit, then the output of the bridge becomes an indication of the temperature difference between the middle and the ends of the waveguide (see Fig. 4). Suppose that the ends of the waveguide along with resistances R_1 and R_4 are at some reference temperature, and the center of the waveguide along with resistances R_2 and R_3 is elevated in temperature by θ degrees Celsius. The ratio of the detector voltage V_d due to the unbalanced bridge to the source voltage E is

$$\frac{V_d}{E} = \frac{k\theta}{2 + k\theta} \qquad (17)$$

where θ is the temperature difference between the center and the ends of the calorimeter, and k is the temperature coefficient of resistance of the wire used.



(a)

(b)

Fig. 4—(a) Location of resistance windings on calorimeter. (b) Bridge circuit.

If both k and θ are relatively small, so that their product is much less than 2, then (17) may be approximated as

$$\frac{V_d}{E} \approx \frac{k\theta}{2} \cdot$$

Combining this with (15) and rewriting gives an expression for the voltage output of the bridge per unit power flowing through the waveguide

$$\frac{V_d}{P_{in}} = \frac{0.0575 l^2 k \alpha E}{aK} \cdot \qquad (18)$$

Eq. (18) may then be used to predict the amount of output signal expected from a given calorimeter.

Experimental Calorimeters

Characteristics

If a resistance-wire bridge is used to indicate the temperature rise in the center of the calorimeter, then (18) and the first three terms of the summation of (8) can be combined to give a reasonably accurate expression for the voltage output of the bridge circuit as a function of bridge voltage input, waveguide power input, and time

$$V_d = \frac{0.0575 l^2 k \alpha E P_{in}}{aK} \left\{1 - 1.032 e^{-t/\tau} + 0.0381 e^{-27t/\tau} - 0.0082 e^{-125t/\tau}\right\} \qquad (19)$$

where $\tau = 4l^2 s\rho/\pi^2 K$ as in (8). Eq. (19) along with Table I can then be used to predict the behavior of a calorimeter constructed using a given waveguide material. There is a slight disadvantage to building a calorimeter using a waveguide of a uniform material in that it is desirable to have a high waveguide attenuation in order to have a reasonable output as well as desirable to have a high thermal conductivity in order to have a short time constant. To examine this, (8), (10) and (15) can be combined and rewritten

$$\theta(0, \infty) = \frac{0.115 l^2 P_{in} \alpha}{aK}$$

$$\tau = \frac{l^2}{2K\eta} \cdot \qquad (20)$$

Thus to have a reasonable center temperature rise and a short time constant it is desirable to select a material with large thermal conductivity and large attenuation. These are, of course, almost mutually exclusive qualities, so the obvious solution is to coat the inside of a high-thermal-conductivity waveguide with a lossy material to give the waveguide a high attenuation. This arrangement is a bit impractical from the experimental standpoint, although for a production device it might be an excellent way of achieving both a reasonable sensitivity and a short time constant.

According to Table I, a calorimeter constructed using stainless steel would be the most sensitive, but it would also have the longest time constant. Of the materials tabulated, copper and stainless steel would be the best combination for a coated waveguide type of construction. For experimental purposes, however, the stainless steel is the most satisfactory. The results obtained using stainless steel can be extrapolated to cover other waveguide materials.

The resistances of the arms of the calorimeter bridge circuit are chosen to minimize the error due to heating the calorimeter from power losses in the bridge circuit itself. The power dissipated in a calorimeter of length $2l$ and of attenuation per unit length α is $0.46 l \alpha P_{in}$, while that dissipated in the bridge is E^2/R. If the power dissipated in the bridge is to be limted to 1–2 per cent of the absorbed waveguide power for the smallest expected waveguide input power, then for a given bridge input voltage the minimum bridge arm resistance is determined.

Two S-band calorimeters using stainless steel waveguide have been designed and tested: a short model with water-cooled flanges and a longer model with water-cooled clamp blocks designed to be built on the existing waveguide of a system. The theoretical and experimental values of sensitivity and time constant have been compared for both models, and the minimum resolution has been measured for the short model.

Short Calorimeter with Flanges

In order to demonstrate experimentally the feasibility of an in-line calorimeter, a model $3\frac{3}{4}$ inches long was constructed using 3.00×1.50 inch $\times 0.065$ inch wall stainless steel waveguide and copper water-circulating vacuum flanges (see Fig. 5). An alloy wire of 70 per cent nickel and 30 per cent iron ("Balco," a trademark of the Wilbur B. Driver Co.) was selected for the resistance bridge windings as having the greatest available temperature coefficient of resistance along with a high tensile strength. The resistances of the bridge windings were selected to be approximately 200 Ω, so that the total bridge generated power is less than 2 per cent of the power absorbed by the calorimeter when the input power is 100 w and the bridge input is 1 v. The resistance windings are 200.0 Ω each at 26°C; a small 1-Ω trimming potentiometer is included to balance the bridge. The entire unit is insulated with foam plastic to reduce convection and radiation heat losses.

The pertinent constants for this calorimeter are as follows:

$l = 3.80$ cm	$K = 1.506 \times 10^{-1}$ w/°C-cm
$a = 3.66$ cm^2	$s = 0.502$ joules/°C-gm
$k = 4.5 \times 10^{-3}$/°C	$\rho = 7.93$ gm/cm^3
$E = 1.0$ v assumed	$\alpha = 0.0469$ db/ft measured at 2.856 Gc with a tuned reflectometer.

Substituting these values in (19) gives

$$V_d = 13.25 P_{\text{in}}(1 - 1.032 e^{-t/156.5} + 0.0384 e^{-t/17.4} - 0.008 e^{-t/1.25})$$

where time t is in seconds, power input P_{in} is in kilowatts, and detector voltage to open circuit V_d is in millivolts. The sensitivity of this calorimeter should then be 13.25 mv out per volt input to the bridge per kilowatt flowing through the calorimeter. The first term of the summation is dominant, so the rise time can be assumed to be controlled by a single time constant, in which case the computed time constant is $\tau = 2$ min, 36.5 sec.

The sensitivity and time constant of the calorimeter were measured at 2.856 Gc in a high-power resonant ring waveguide test setup. Average power levels up to 1 kw were used to determine the sensitivity, while the time constant was determined by measuring the decay of the bridge output voltage with time after the exciting power in the ring circuit was turned off. Under the assumption that the rise and fall times of the calorimeter are dominated by a single exponential, the time constant was taken as being the time necessary for the bridge output to decrease to $1/e$ of its steady-state excited value. The calibration of the ring circuit power monitor was accurate to 10 per cent, so any experimental figure for sensitivity may be inaccurate by 10 per cent. The experimentally observed sensitivity was 12.5 ± 1.25 mv out per volt into bridge per kilowatt flowing through the calorimeter while the time constant was observed to be 2 min, 30 sec. This model of the calorimeter was also checked for minimum resolution using a low-power signal generator. The ambient air temperature was maintained at $25° \pm 2°$C, while the flange cooling water was maintained at a temperature of 18.5° ± 0.6°C. The microwave power through the calorimeter was cycled: 0.9 w average for one-half hour, zero power for one-half hour. The recording of the bridge output is shown in Fig. 6. As can be seen from the recording, the instabilities in the calorimeter output are equivalent to no more than about 0.5 w of power transmitted through the device. It might then be concluded that the ultimate resolution of the calorimter is 0.5 w.

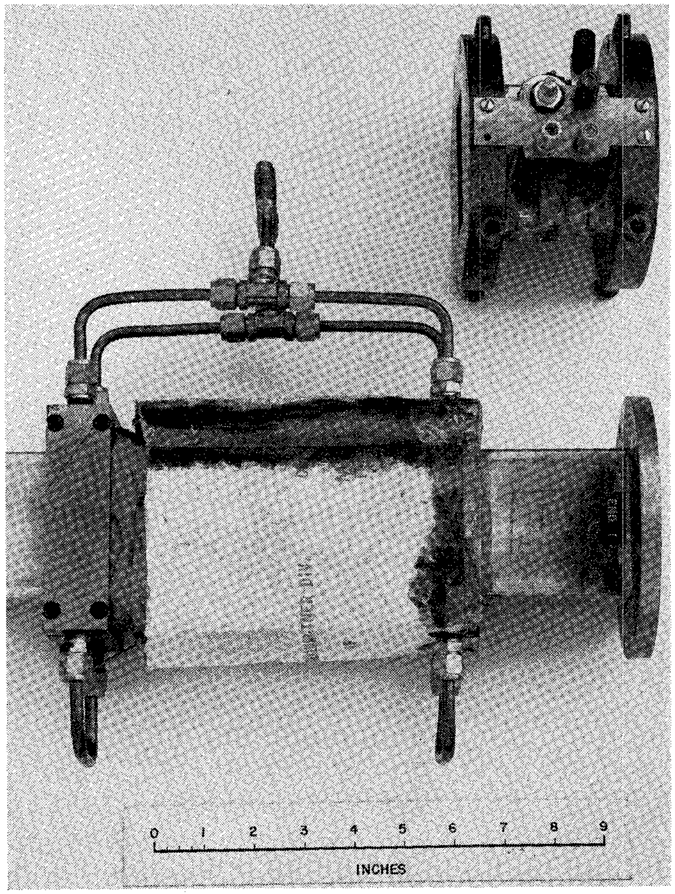


Fig. 5—Short calorimeter with flanges.

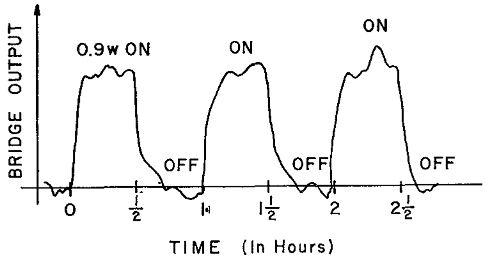


Fig. 6—Minimum resolution test recording.

Clamp-On Calorimeter One Waveguide-Wavelength Long

To illustrate the simplicity of the in-line calorimeter, a model with clamp-on heat sinks was constructed (see Fig. 5). This is distinctly different from the previous model in that it can use an existing waveguide and does not necessarily require flange connections.

In order to minimize the error in reading the true sum of the powers passing through the calorimeter, the half-length l should be a multiple of half waveguide wavelengths. The total length of the shortest calorimeter possible is then one waveguide wavelength. The waveguide wavelength at the test frequency of 2.856 Gc in the stainless steel waveguide was measured by sliding a short through the waveguide positioned in the output arm of a detuned three-arm reflectometer, and was found to be 14.98 cm. Water cooling blocks were clamped 14.98 cm apart on a length of stainless steel waveguide to form the calorimeter. The bridge resistances are 380 Ω, so that the maximum bridge power for an input waveguide power of 100 w and a bridge input of 1 v will be only 1.5 per cent of the power absorbed by the waveguide. Other than the length and the arm resistances, the constants for this model of the calorimeter are identical to those of the short calorimeter. The expression for the output voltage of the bridge of this calorimeter is then

$$V_d = 51.5P_{\text{in}}(1 - 1.032e^{-t/600} + 0.0384e^{-t/66.7} - 0.00826e^{-t/4.8}).$$

The sensitivity of this model is then 51.5 mv output per volt input per kilowatt flowing through the calorimeter. The first term of the summation is again dominant, so the time constant can be taken as $\tau = 10$ min. The sensitivity and time constant were measured in the same ring waveguide test setup as used for the previous model. Average power levels up to 2450 w were used to determine the sensitivity, while the time constant was determined in the same fashion as was that for the short model. The experimental sensitivity was 47 ± 4.7 mv output per volt input to bridge per kilowatt flowing through the calorimeter while the time constant was observed to be 11 min.

Conclusions

The close agreement between the theoretical and experimental values of sensitivity and time constant for the two experimental calorimeters illustrates the utility of the in-line calorimeter in that it is not only a simple device but also has predictable characteristics. These results seem to indicate that if a calorimeter were made with extreme care, insulated as well as possible, and supplied with a constant-temperature, constant-flow-rate coolant to its heat sinks, it could easily serve as a secondary high-power standard. The chief disadvantage of the in-line calorimeter is its inherently long time constant: ten minutes for the longer calorimeter may well be far too long for most applications. If, however, the longer calorimeter were constructed of copper waveguide with a coating of stainless steel on the inside, (20) indicates that the time constant would be 21 sec, while the sensitivity would be 2.02 mv output per volt input per kilowatt flowing through the calorimeter.

It may then be concluded that the inherently simple in-line calorimeter is potentially an accurate device which can be made to have a reasonable time constant while maintaining an adequate sensitivity.

Acknowledgment

The author would like to acknowledge the valuable assistance of Dr. K. B. Mallory of the Stanford Linear Accelerator Center and Dr. C. F. Quate of the Microwave Laboratory, W. W. Hansen Laboratories of Physics, at Stanford University. In addition, the interest shown by R. W. Beatty and G. F. Engen of the National Bureau of Standards Boulder Laboratories is deeply appreciated. The experimental portion of this work was facilitated through the use of equipment designed by J. Jasberg of the Stanford Linear Accelerator Center.